AFCRC-TN-60-392

NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

RESEARCH REPORT No. EM-154

Radio Propagation Past a Pair of Dielectric Interfaces

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Contract No. AF 19(604)5238

MAY, 1960

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The research reported in this document has been sponsored by the Electronics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, under Contract No. AF 19(604)5238, and by the American Petroleum Institute.

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Abstract

In a previous report we have introduced a linear boundary condition that serves to accurately replace transition conditions at dielectric-dielectric interfaces. In this work we apply this procedure to obtain an approximate solution for an otherwise mathematically intractable problem. The original geometry of the problem is that of a dielectric half space above two dielectric quarter spaces. After we apply our technique of re-formulation, the problem reduces to one of obtaining a solution of a two-part boundary value problem, in the upper half space. This problem is solved exactly by the method of Wiener and Hopf. Physically reasonable results are obtained in a form suitable for numerical computation.

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Introduction

In Part I we have discussed a procedure which allows one to replace a dielectric-dielectric interface by a linear boundary condition. We have made this approach plausible by: (1) proving that it guarantees at most a small error in the far field of a line source above a dielectric half space, (2) proving reciprocity and uniqueness theorems for this geometry, and (3) obtaining excellent agreement in a comparison of the use of this formulation with an exact solution in a problem involving diffraction. However, we have not demonstrated the use of this approach in a hitherto unsolved problem.

In this work we seek to find the field of the following problem: A plane wave is incident in a dielectric half space above two dielectric wedges (cf. Figure 1). As the problem stands it is not amenable to available mathematical techniques. However, we have made plausible a procedure which replaces a dielectric-dielectric interface by a linear boundary condition. This then allows us to replace two of the dielectric interfaces shown in Figure 1 by two different linear boundary conditions of the form described in Part I (cf. Figure 2). We neglect the phenomena arising at the interface between the two wedges in the lower half space.

In Section 1 we explicitly formulate the problem shown in Figure 2, and obtain an exact solution in Section 2. We devote Section 3 to an analysis of the field in a vicinity of the origin. We find that the solution has reasonable behavior in that neighborhood. In Section 4 we obtain asymptotic expressions which describe the field for large distances from the origin. We conclude by compiling the results in Section 5 in a form suitable for computation.

1. Formulation

In this report we seek an explicit approximate solution to the following non-separable problem:

A plane wave $e^{ik(x \cos \theta_o - y \sin \theta_o)}$ is incident upon two dielectric quarter-spaces (cf. Figure 1) at some angle θ_o.

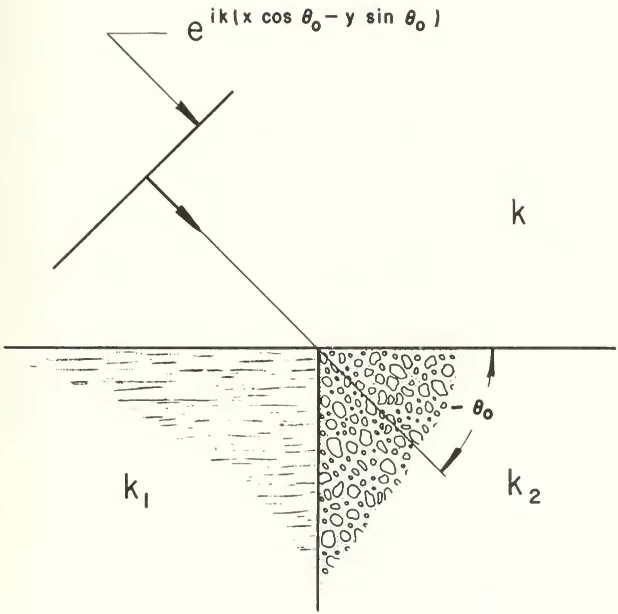

Figure 1

We seek solutions of the wave equations

$$(1) \qquad \begin{cases} (\nabla^2 + k^2)\, u(x,y) = 0 & y \geq 0\,; \\ (\nabla^2 + k_1^2)\, u(x,y) = 0 & y \leq 0,\ x \leq 0 \\ (\nabla^2 + k_2^2)\, u(x,y) = 0 & y \leq 0,\ x \geq 0 \end{cases}$$

where

$$(2) \qquad \begin{cases} k < k_1 \\ k < k_2 \end{cases}$$

and appropriate continuity conditions are to be satisfied at each interface. These boundary conditions are derived from the physics of the problem: In our context $u(x,y)$ represents one of the transverse components of the electromagnetic field, either H_z or E_z. If $u = H_z$ and $H_x = H_y = 0$ then we say that we have transverse magnetic or TM excitation. Correspondingly, if $u = E_z$, $E_x = E_y = 0$ then we shall speak of transverse electric or TE excitation. In either case solving the problem for $u(x,y)$ yields the remaining components by use of the source-free Maxwell equations

$$(3) \qquad \nabla \times \vec{E} = i\omega\mu\vec{H}$$

$$(4) \qquad -\nabla \times \vec{H} = i\omega\varepsilon\vec{E}$$

with a suppressed time factor of $e^{-i\omega t}$.

The boundary conditions referred to above are determined by the continuity of the following components

$$\vec{\nu} \times \vec{E} \quad \text{and} \quad \vec{\nu} \cdot \vec{B}$$

of the electromagnetic field across a discontinuity in k [6, p. 37]. The vectors $\vec{\nu}$ and $\vec{s}$ are the unit normal and tangent vector at each interface. The results of Part I of this work lends plausibility to the conclusion that we can replace these continuity conditions by a linear boundary condition of the form

$$\frac{1}{ik} \frac{\partial}{\partial \vec{\nu}} + A + \frac{B}{k^2} \frac{\partial^2}{\partial s^2} u = 0 \quad . \tag{5}$$

In the sequel we shall illustrate the procedure for a transverse magnetic (TM) excitation. For this problem we choose the coefficients A and B in the boundary condition to give an exact match at normal incidence and Brewster's angle incidence. It is convenient to note that

(6) A and B are real and positive

and

(7) $A > B$.

Both of these requirements are consistent with the results of Part I.

With this information we can replace the three-media problem (1) for $y \geq 0$ by the following two-part boundary-value problem which can be solved explicitly by the method of Wiener and Hopf (cf. Figure 2).

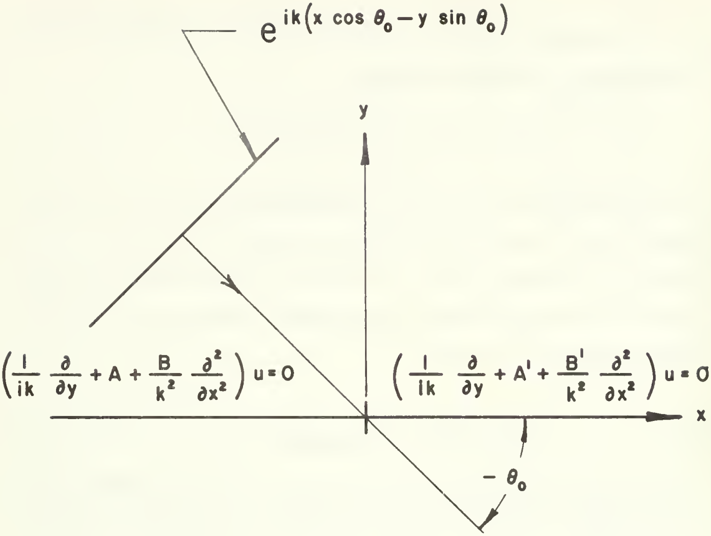

Figure 2

(8) $$u_{inc} = e^{ik(x\cos\theta_0 - y\sin\theta_0)}$$

(9) $$(\nabla^2 + k^2)\, u(x,y) = 0, \qquad y \geq 0$$

(10) $$\left(\frac{1}{ik}\frac{\partial}{\partial y} + A + \frac{B}{k^2}\frac{\partial^2}{\partial x^2}\right) u = 0, \qquad y = 0,\ x < 0$$

(11) $$\left(\frac{1}{ik}\frac{\partial}{\partial y} + A' + \frac{B'}{k^2}\frac{\partial^2}{\partial x^2}\right) u = 0, \qquad y = 0,\ x > 0 \ .$$

We seek a solution of the form

$$u(x,y) = u_{inc}(x,y) + u_s(x,y) \quad . \tag{12}$$

and add the condition that $u_s(x,y)$ and its first derivatives be bounded and continuous in any finite neighborhood of the origin. In Section 3 this condition will be shown to be sufficient to guarantee that the origin behaves neither as a source nor a sink. The solution of this problem will occupy our attention for the balance of the report.

2. Solution

Introduce the complex ν-plane

$$\nu = \rho\, e^{i\theta}$$

in which we define the radical

$$R(\nu) = \sqrt{k^2 - \nu^2} \tag{1}$$

as follows: Assume k to have a vanishingly small imaginary part ε

$$k = |k|\; e^{i\varepsilon}$$

and draw the branch cuts from $+k$ and $-k$ to infinity along the rays $\theta = \varepsilon$ and $\theta = \pi + \varepsilon$ respectively (cf. Figure 3). If we choose

$$\nu - k = \rho^+\, e^{i\theta^+} \;, \quad -2\pi + \varepsilon \leq \theta^+ \leq \varepsilon$$

$$\nu + k = \rho^-\, e^{i\theta^-} \;, \quad -\pi + \varepsilon \leq \theta^- \leq \pi + \varepsilon$$

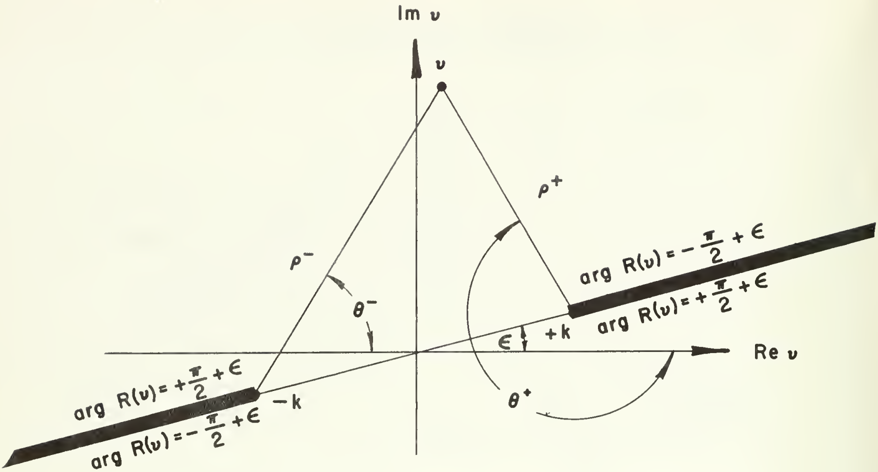

Figure 3

then $R(\nu)$ is uniquely determined throughout the cut ν-plane by choosing

$$\arg R(\nu) = \frac{1}{2}(\pi + \theta^+ + \theta^-)$$

for example

$$R(0) = |k|\, e^{i\frac{\pi}{2}}\, e^{-\frac{1}{2}(\pi - \varepsilon)i}\, e^{\frac{1}{2}\varepsilon i}$$

(2)

$$= |k|\, e^{\varepsilon i} = +k$$

and, in the same manner, $\arg R(\nu) = -\frac{\pi}{2} + \varepsilon$ immediately above the branch cut at $\nu = +k$, and $\arg R(\nu) = \frac{\pi}{2} + \varepsilon$ immediately below that cut. Since $R(\nu)$ is even in ν this also fixes its values about the cut in the lower ν-plane.

A virtue of this choice of branch cuts is that no zeros of

$$Z(\nu) \equiv \frac{\sqrt{k^2 - \nu^2}}{k} + A - \frac{B\nu^2}{k^2} \tag{3}$$

or

$$Z'(\nu) \equiv \frac{\sqrt{k^2 - \nu^2}}{k} + A' - \frac{B'\nu^2}{k^2} \tag{4}$$

can arise in the cut ν-plane.*

A solution of the wave equation (1.1) can be obtained by separation of variables in the form

$$u_s(x,y) = \frac{1}{2\pi i} \int_{-\infty + i0}^{\infty + i0} A^-(\nu) \frac{e^{i\nu x + i\sqrt{k^2 - \nu^2}\, y}}{\frac{1}{k}\sqrt{k^2 - \nu^2} + A - \frac{B\nu^2}{k^2}} \, d\nu \tag{5}$$

where the path of integration is along the real ν-axis. $A^-(\nu)$ is some unknown function of ν of algebraic growth (unless otherwise specified, when speaking of the order of a function we shall always mean the order of

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*See Appendix C of Part I.

growth of that function at infinity $|\nu| \to \infty$). If $A^-(\nu) = O(\nu^{\varepsilon_1})$, where $\varepsilon_1 < 0$ and $A^-(\nu)$ is analytic for Im $\nu \leq 0$, then the function (5) will satisfy the boundary condition (1.2) by Jorden's lemma, if we assume that we can differentiate freely beneath the integral sign. We shall construct a solution to the problem (1.8)-(1.12) in terms of this secondary field $u_s(x,y)$ and the incident plane wave as well as geometrically reflected plane waves.

Let $R(\theta_o)$ and $R'(\theta_o)$ be the reflection coefficients such that $u_{inc} + Ru_{ref}$ or $u_{inc} + R'u_{ref}$ satisfy the left-hand (1.10) or eight hand (1.11) boundary condition respectively where

$$u_{ref} = e^{ik(x \cos \theta_o + y \sin \theta_o)}$$

$$u_{inc} = e^{ik(x \cos \theta - y \sin \theta_o)}$$

The construction u_T identified with the total field

$$u_T(x,y) = u_{inc}(x,y) + Ru_{ref}(x,y) + u_s(x,y)$$

will then also satisfy the boundary condition (1.10).

In order to choose $A^-(\nu)$, we note that

$$u_{inc} + Ru_{ref} = u_{inc} + R'u_{ref} + (R - R')\, u_{ref}$$

so that after application of the boundary condition (1.11) at $y = 0$ to u_T we are left with

$$(6) \qquad \alpha(R-R')\, e^{ikx\cos\theta_o} + \frac{1}{2\pi i}\int_{-\infty+i0}^{\infty+i0} K(\nu)\, A^-(\nu)\, d\nu$$

where

$$(7) \qquad K(\nu) = \frac{k\sqrt{k^2-\nu^2} + k^2A' - B'\nu^2}{k\sqrt{k^2-\nu^2} + k^2A - B\nu^2}$$

and

$$(8) \qquad \alpha = Z'(k\cos\theta_o) = \sin\theta_o + A' - B'\cos^2\theta_o \quad .$$

Since for $k = |k|e^{i\varepsilon}$ we have

$$e^{ikx\cos\theta_o} = \frac{1}{2\pi i}\int_{-\infty+i0}^{\infty+i0} \frac{e^{i\nu x}}{\nu - k\cos\theta_o}\, d\nu$$

the expression (6) will vanish by Jordan's lemma if $G^+(\nu)$

$$(9) \qquad G^+(\nu) = \frac{\alpha(R-R')}{\nu - k\cos\theta} + K(\nu)\, A^-(\nu)$$

is an analytic function for $\mathrm{Im}\,\nu \geq 0$ of algebraic order $O(\nu^{\varepsilon_2})$ where ε_2 is any negative number. The problem will be solved once the required $A^-(\nu)$ and $G^+(\nu)$ are found by an appeal to Liouville's theorem

For this purpose we show in appendix A that $K(\nu)$ can be expressed as

$$(10) \qquad K(\nu) = P^+(\nu)\, P^+(-\nu)$$

where $P^+(\nu)$ is analytic, and zeroless for $\mathrm{Im}\,\nu \geq -\mathrm{Im}\,k$ and $O(1)$ at infinity.

We re-write (9) as

$$(11) \quad \frac{G^{+}(\nu)}{P^{+}(\nu)} + \frac{\alpha(R - R')}{\nu - k\cos\theta_o}\left[\frac{1}{P^{+}(k\cos\theta)} - \frac{1}{P^{+}(\nu)}\right] = \frac{\alpha(R - R')}{\nu - k\cos\theta_o}\frac{1}{P^{+}(k\cos\theta_o)}$$

$$+ A^{-}(\nu)\, P^{+}(\nu) \quad .$$

Owing to the assumed behavior of $A^{-}(\nu)$ and $G^{+}(\nu)$ the left side of (11) is analytic for $\operatorname{Im} \nu \geq 0$ and the right side analytic for $\operatorname{Im} \nu \leq 0$, thus each is the analytic continuation of the other and hence, defines an entire function.

Furthermore, each side is of order $O(\nu^{\varepsilon_3})$, where $\varepsilon_3 = \max(\varepsilon_1, \varepsilon_2)$ is less than zero, so that each side of (11) is equal to zero by an application of Liouville's theorem; this allows one to solve for both $A^{-}(\nu)$ and $G^{+}(\nu)$.

$$(12) \qquad A^{-}(\nu) = \frac{\alpha(R - R)}{(\nu - k\cos\theta_o)\, P^{+}(k\cos\theta)}\ \frac{P^{+}(\nu)}{K(\nu)}$$

and

$$(13) \qquad G^{+}(\nu) = \frac{\alpha(R' - R)}{\nu - k\cos\theta_o}\left[\frac{P^{+}(\nu)}{P^{+}(k\cos\theta)} - 1\right]$$

The complete solution to the problem can then be displayed as

$$(14) \quad u_{inc} + Ru_{ref} + \frac{\alpha(R' - R)}{2\pi i\, P^{+}(k\cos\theta)} \int_{-\infty}^{\infty} \frac{P^{+}(\nu)\, e^{i\nu x + i\sqrt{k^2 - \nu^2}\, y}\, d\nu}{(\nu - k\cos\theta_o)\left(\frac{\sqrt{k^2 - \nu^2}}{k} + A' - \frac{B'}{k^2}\nu^2\right)} \, .$$

3. The Near Field

We consider the integral representing the scattered field

$$(1) \quad u_s(x,y) = \gamma \int_{-\infty}^{\infty} \frac{P^+(\nu)\, e^{i\nu x + i\sqrt{k^2-\nu^2}\, y}}{(\nu - k\cos\theta_o)\left(\frac{\sqrt{k^2-\nu^2}}{k} + A' - \frac{B'}{k^2}\nu^2\right)}\, d\nu$$

where

$$(2) \quad \gamma = \frac{(R' - R)\alpha}{2\pi i\, P^+(k\cos\theta_o)}$$

and observe that apart from the exponential factor the integrand is $O(\nu^{-3})$ and consequently the integral continues to converge for $x = y = 0$. Indeed, any linear combination of first derivatives with respect to x and y of $u_s(x,y)$ will also be bounded and continuous in any neighborhood of the origin. The implication of these last remarks is that the origin neither absorbs nor emits radiation.

To clarify this point we observe that if we identify $u(x,y)$ as the transverse magnetic component of the electromagnetic field $u = H_z$, $H_x = H_y = 0$, then Maxwell's equation*

$$\nabla \times \vec{H} = -i\omega\varepsilon\, \vec{E}$$

in cylindrical coordinates (ρ, θ, z)

$$x = \rho\cos\theta$$

$$y = \rho\sin\theta$$

*Having obtained a solution, we make the customary transition Im k = 0 in the rest of this section.

becomes

$$- i\omega\varepsilon \quad E_\rho = \frac{1}{\rho}\frac{\partial u}{\partial\theta}$$

$$- i\omega\varepsilon \quad E_\theta = - \frac{\partial u}{\partial\rho}$$

so that (1) can be re-written as

$$u_s(\rho,\theta) = \gamma \int_{-\infty}^{\infty} N(\nu)\, e^{ik\rho\,(\nu\cos\theta + \sqrt{k^2-\nu^2}\sin\theta)} d\nu$$

where

$$N(\nu) = O(\nu^{-3}), \qquad |\nu| \to \infty$$

hence

$$E_\rho = \frac{ik\gamma}{-i\omega\varepsilon} \int_{-\infty}^{\infty} (-\nu\sin\theta + \sqrt{k^2-\nu^2}\cos\theta) N(\nu) e^{ik\rho(\nu\cos\theta + \sqrt{k^2-\nu^2}\sin\theta)}$$

and

$$E_\theta = \frac{ik\gamma}{-i\omega\varepsilon} \int_{-\infty}^{\infty} (\nu\cos\theta + \sqrt{k^2-\nu^2}\sin\theta) N(\nu) e^{ik\rho(\nu\cos\theta + \sqrt{k^2-\nu^2}\sin\theta)}\, d\nu \quad .$$

Both E_ρ and E_θ are bounded and continuous in any neighborhood of the origin. Consequently, if we form the surface integrals over the Poynting flux

$$P = \int_S (\vec{E} \times \vec{H}^*) \cdot d\vec{\sigma} = \int_{S'} E_\theta\, u\, |d\vec{\sigma}|$$

about the surface S consisting of a cylinder of unit height along the z-axis whose base is a semicricle of radius ρ in the xy-plane

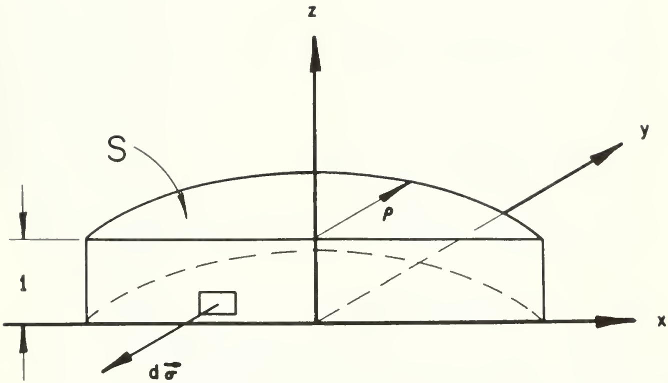

Figure 4

then as $\rho \to 0$, $P \to 0$, since each term in the integrand of P is bounded about $\rho = 0$.

4. The Far Field

In this section we will obtain an asymptotic development of the integral

$$(1) \qquad u_s(x,y) = \gamma \int_{-\infty}^{\infty} \frac{P^+(\nu)\, e^{i\nu x + i\sqrt{k^2-\nu^2}\, y}}{(\nu - k\cos\theta_0)\, Z'(\nu)}\, d\nu$$

where we recall the definitions

(2) $$\gamma = \frac{(R' - R)}{2\pi i \; P^{+}(k \cos \theta_o)}$$

(3) $$\alpha = Z'(k \cos \theta_o) = \sin \theta_o + A' - B' \cos^2 \theta_o$$

and

(4) $$Z'(\nu) = \frac{\sqrt{k^2 - \nu^2}}{k} + A' - \frac{B'\nu^2}{k^2} \quad .$$

It is convenient to make the transformations

(5) $$\nu = k \cos \phi$$

and

(6) $$x = \rho \cos \theta$$

$$y = \rho \sin \theta$$

so that (1) becomes

(7) $$u_s(\rho,\theta) = -\gamma \int_C \frac{P^{+}(k \cos \phi)\, e^{ik\rho \cos (\phi - \theta)} \sin \phi \, d\phi}{(\cos \phi - \cos \theta_o)\, Z'(k \cos \phi)}$$

where the contour C is taken as shown in Figure 5. Note that the transformation (5) has the effect of removing the branch cuts at $\nu = \pm k$.

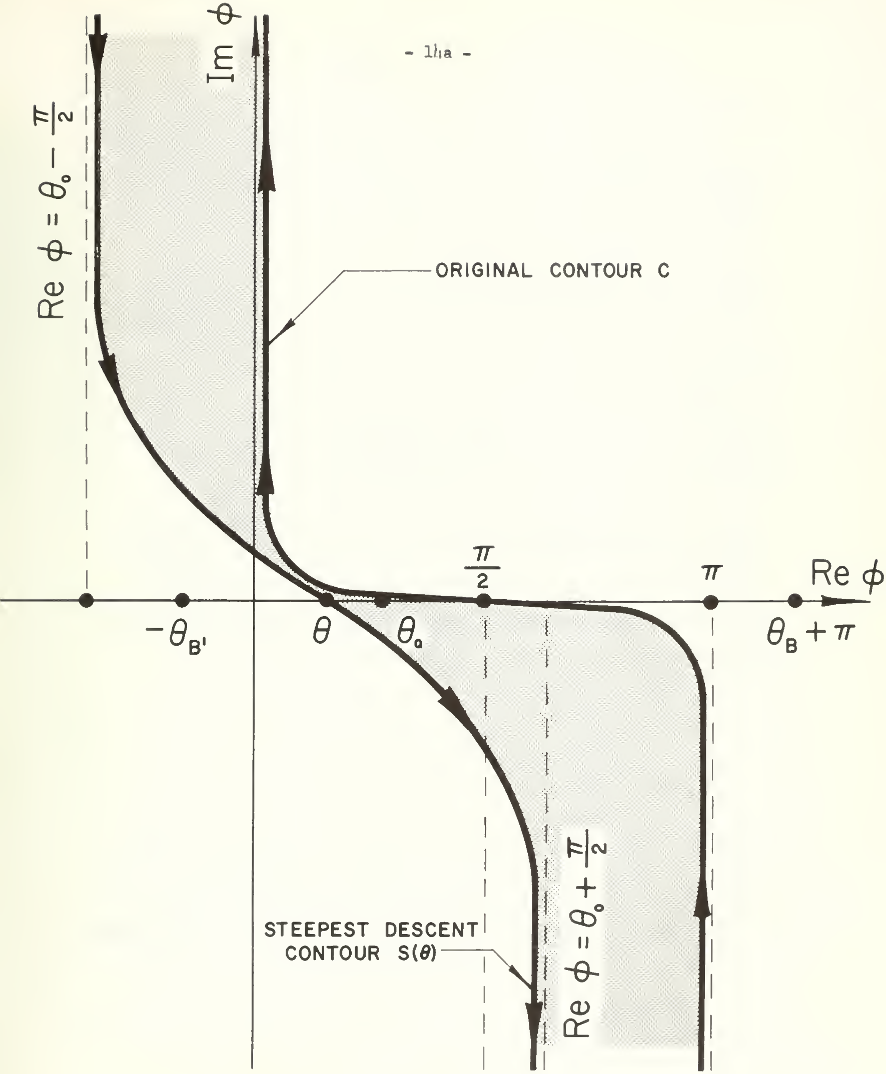
Im ϕ
Re $\phi = \theta_0 - \frac{\pi}{2}$
ORIGINAL CONTOUR C
$\frac{\pi}{2}$
π
Re ϕ
$-\theta_{B'}$
θ
θ_0
$\theta_B + \pi$
Re $\phi = \theta_0 + \frac{\pi}{2}$
STEEPEST DESCENT
CONTOUR S(θ)

Figure 5

Following standard operating procedure we introduce the steepest descents contour $S(\theta)$. This path is defined as the locus of points such that

$$(8) \qquad \cos(\phi - \theta) = 1 + is^2$$

or

$$(9) \qquad s = +\sqrt{2}\, e^{\pi i/4} \sin\left(\frac{\phi - \theta}{2}\right)$$

where s is any real number $-\infty \leq s \leq +\infty$. For fixed θ it originates at $\phi = (\theta - \pi/2, +i\infty)$, crosses the real axis at θ at an angle $\pi/4$, and terminates at $\phi = (\theta + \pi/2, -i\infty)$. The virtue of this contour is that the exponential $e^{ik\rho \cos(\phi - \theta)}$ becomes

$$e^{ik\rho} \cdot e^{-k\rho s^2}$$

along this path. Hence if we can transform C to $S(\theta)$ then the major contribution to the integral (7) arises from a neighborhood of $s = 0$. If there are no singularities of the integrand of (7) for a sufficiently

large neighborhood of $s = 0$, or $\emptyset = \theta$ we can obtain a good approximation to u_s by expanding

$$(10) \qquad K(\emptyset) = \frac{P^+(k \cos \emptyset)}{(\cos \emptyset - \cos \theta_o)\, Z(\theta)}$$

as a power series about $\emptyset = \theta$, or $s = 0$ and integrating (7) term by term. We see from (10) that the pertinent singularities are at $\emptyset = \theta_o$ and any zeros of $Z(\theta)$. The procedure of Van der Waerden is to isolate these poles and discuss them separately. The singularity at $\emptyset = \theta_o$ will be shown to generate a function which affords the transition across the shadow boundary. Roots of $Z(\nu)$ will be shown to be important if our direction of observation θ is near the interface.

We shall illustrate our remarks for the TM case for which $u(x,y)$ represents the magnetic component H_z. If $u = H_z$, then from (4)

$$(11) \qquad Z'(-\emptyset) = -\cos \emptyset + (A' - B' \sin^2 \emptyset)$$

and by the discussion of Part I we have chosen A' and B' so that

$$(12) \qquad Z'(-\theta_{B'}) = 0$$

where $\theta_{B'}$ is the Brewster's angle of the $k - k_2$ interface

$$(13) \qquad \theta_{B'} = \tan^{-1} \frac{k_2}{k} \quad .$$

If we define

$$Z(\phi) = \sin\phi + A - B\,k^2 \cos^2\phi$$

and use (2.10) then we have an alternate representation for (7) as

$$(14) \quad u_s(\rho,\theta) = -\gamma \int_C \frac{e^{ik\rho\cos(\phi-\theta)}\sin\phi\,d\phi}{P^+(-k\cos\phi)\,Z(k\cos\phi)\,(\cos\phi-\cos\theta_o)}$$

The representation (7) will be useful for an asymptotic development in the range $0 \leq \theta \leq \pi/2$ while the representation (14) is preferred for $\pi/2 \leq \theta \leq \pi$. The same procedure that derived (12) will yield

$$(15) \qquad Z(\theta_B - \pi) = 0$$

where θ_B is the Brewster angle for the k-k_1 interface

$$(16) \qquad \theta_B = \tan^{-1}\frac{k_1}{k} \quad .$$

Collecting these results we see that $-\theta_B$, is in the neighborhood of $\phi = 0$ if $k_2 \gg k$ and $-\pi + \theta_B$ is in the vicinity of $\phi = \pi$ for $k_1 \gg k$. If $k_2 \gg k$ we shall see that the singularity of the integrand in (7) at $\phi = \theta$ will influence the asymptotic development of $u_s(\rho,\theta)$ for θ in a neighborhood of $\theta = 0$. Similarly if $k_1 \gg k$ the development of $u_s(\rho,\theta)$ will be influenced for a vicinity of $\theta = \pi$.

We can use the preceding ideas in the following manner: Let

$G(\phi)$ be analytic in a sufficiently wide circle about the saddle point $\phi = \theta$. Then we can expand $G(\phi) = \widetilde{G}(s)$ in the s-plane about $s = 0$ as

$$(17) \qquad \widetilde{G}(s) = G(\theta) + g_1 s^2 + \dots + g_{2n} s^{2n} + \dots$$

and the integral

$$I(\theta) = \int_{S(\theta)} G(\phi)\, e^{ik\rho \cos(\phi - \theta)}\, d\phi = e^{ik\rho} \int_{-\infty}^{\infty} \widetilde{G}(s)\, \frac{e^{-k\rho s^2} ds}{\cos\left(\frac{\phi - \theta}{2}\right)}$$

can be integrated term by term to yield

$$(18) \qquad \boxed{I(\theta) = \sqrt{\frac{2\pi}{k\rho}}\; e^{i(k\rho - \pi/4)}\, G(\theta) + O\left[(k\rho)^{-3/2}\right] ,}$$

the conventional saddle point result. However* if $\widetilde{G}(s)$ has a singularity in the immediate vicinity of $s = 0$ then the radius of convergence of (17) may be too small to permit term by term integration, followed by use of the first term. Let $G(s)$ have a pole at s_o where s_o is in a vicinity of $s = 0$. Then we need to modify the procedure:

We can define $h(s)$ by

$$(19) \qquad \frac{\widetilde{G}(s)}{\cos\left(\frac{\phi - \theta}{2}\right)} = \frac{h(s)}{s - s_o}$$

and then in the manner of Van der Waerden we split off the pole by using the

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*cf. pp. 15, 16 above.

identity

$$(20) \qquad \frac{h(s)}{s - s_o} = \frac{h(s_o)}{s_o} \quad \frac{s}{s - s_o} \left[\frac{h(s) - h(s_o)}{s - s_o} - \frac{h(s_o)}{s_o} \right] .$$

The bracketed term in (20) leads to an integral which can be evaluated without complication by the saddle point method to yield the same result as (18). The other term then leads to the integral

$$(21) \qquad J = \frac{h(s_o)}{s_o} \; e^{ik\rho} \cdot \int_{-\infty}^{\infty} \frac{s}{s - s_o} \; e^{-k\rho s^2} \, ds$$

Van der Waerden shows that this integral may be evaluated in terms of the error function erf(z). The final result is

$$(22) \qquad J = \frac{h(s_o)}{s_o} \; e^{ik\rho} \left\{ \sqrt{\pi/k\rho} - 2\pi i \; s_o \; e^{-k\rho s_o^2} \left[1 - \mathrm{erf}(is_o\sqrt{k\rho}) \right] \right\}$$

For large values of $|s_o^2 \, k\rho|$ one may show that the integral J is $O[(k\rho^{-3/2})]$ and hence can be neglected in comparison with the leading term of (18). Now $G(\phi)$ has poles at $\theta = \theta_o$, $\theta = -\theta_B{}'$, and $\theta = -\pi + \theta_B$. Each of these lead to an s_o by the relation (9). Consequently for each pole say θ_o we will have a region of ρ, θ space exterior to the parabola

$$(23) \qquad |s_o^2 \, k\rho| = |1 - \cos(\theta - \theta_o)| \; k\rho = K \gg 1 \quad \text{say,}$$

for which we can neglect the contribution of (22). The curve

(24) $\quad 1 - \cos(\theta - \theta_o) = K/k\rho$

is a parabola symmetrical about the line $\theta = \theta_o$. Hence, (anticipating our later results somewhat), we will have three parabolas in ρ,θ space in whose interior we need include the term (22). One of these parabolas will contain a transition field across the shadow boundary, and the other two will be centered about the rays $\theta = \theta_B - \pi$, and $\theta = -\theta_{B'}$ where θ_B and $\theta_{B'}$ are the Brewster's angles (cf. Figure 6) for the right and left half-planes.

It is now a simple matter to collect these ideas and obtain an explicit asymptotic development of (7). The first step is to deform the original contour C to the steepest descents $S(\theta)$; in so doing we must pick up the residues at any pole in the region bounded by C and $S(\theta)$. Since the observation space $y \geq 0$ corresponds to $0 \leq \theta \leq \pi$ we never pick up the poles at $\theta_B - \frac{\pi}{2}$ or $\theta_{B'} + \frac{3\pi}{2}$ which both occur outside this range. However if $0 \leq \theta \leq \theta_o$ then we do pick up the residue of the pole at θ_o which corresponds to a transition across the shadow boundary. Using (2), (3), and (7) it is a simple matter to show that the contribution of this residue is

$$(R' - R)\, e^{ik\rho \cos(\phi - \theta_o)}$$

or in the notation of Section 2

(25) $$(R' - R)\, u_{ref} \quad .$$

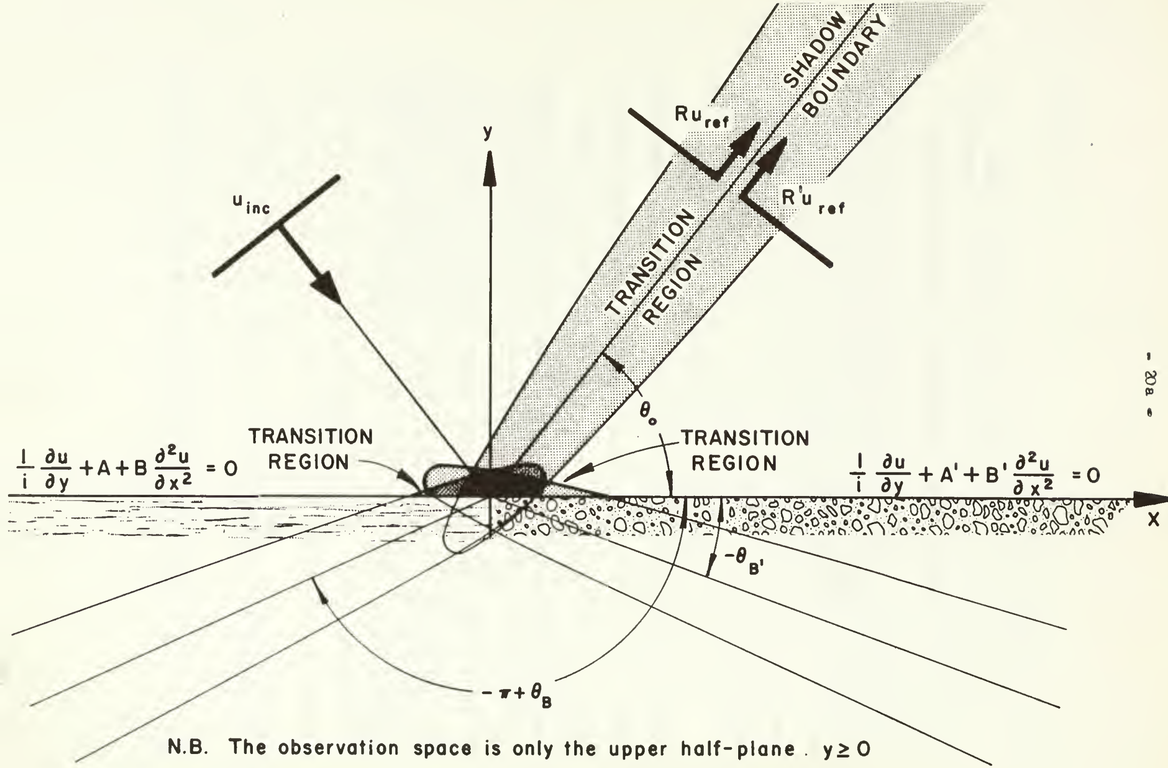
y
u_{inc}
Ru_{ref}
$R'u_{ref}$
SHADOW
BOUNDARY
TRANSITION
REGION
θ_o
TRANSITION
REGION
TRANSITION
REGION
$\frac{1}{i}\frac{\partial u}{\partial y} + A + B\frac{\partial^2 u}{\partial x^2} = 0$
$\frac{1}{i}\frac{\partial u}{\partial y} + A' + B'\frac{\partial^2 u}{\partial x^2} = 0$
X
$-\theta_{B'}$
$-\pi + \theta_B$
N.B. The observation space is only the upper half-plane . $y \geq 0$

Figure 6

If we introduce this result in (2.14), we see that this residue gives us the correct geometrically reflected plane wave (Figure 7) in the total field for the region $\theta_o \leq \theta \leq 0$. The transition across the shadow boundary at $\theta = \theta_o$ is then afforded by a field of the form (22). That is, if our direction of observation is in a vicinity of θ_o and if

$$k\rho \leq \frac{K}{1 - \cos(\theta - \theta_o)}$$

we must include the field (22) where s_o is given in terms of θ_o by (9).

It is worthwhile to observe that the field on the shadow boundary is an elementary function to terms of order $O\left[(k\rho)^{-1/2}\right]$, namely

$$u(\rho, \theta_o) = \frac{R + R'}{2} e^{ik\rho} + O\left[(k\rho)^{-1/2}\right] .$$

Observe that for $k\rho \gg 1$ the field on the shadow boundary approaches the arithmetic mean of the two reflected fields.*

With this information we can draw three parabolas

$$\left.\begin{array}{ll} \text{(a)} & 1 - \cos(\theta - \theta_o) = \dfrac{K}{k\rho} \\[2em] \text{(b)} & 1 - \cos(\theta - \theta_B + \pi) = \dfrac{K}{k\rho} \\[2em] \text{(c)} & 1 - \cos(\theta + \theta_{B'}) = \dfrac{K}{k\rho} \end{array}\right\} \qquad K \text{ fixed}$$

then for fixed $\theta \neq \theta_o$, we can choose $k\rho$ so that if

*This is the result one would expect from experience with related problems.

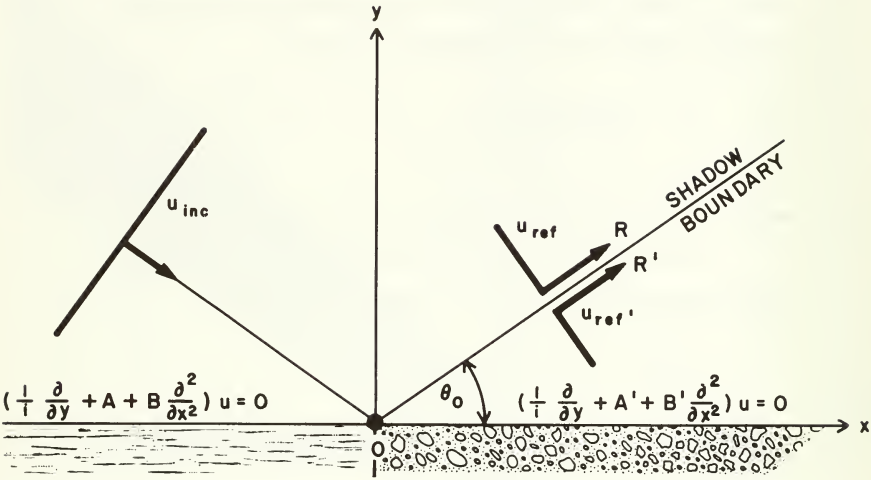
y
u_{inc}
u_{ref}
R
R'
$u_{ref'}$
SHADOW
BOUNDARY
θ_0
$\left(\frac{1}{i}\frac{\partial}{\partial y} + A + B\frac{\partial^2}{\partial x^2}\right)u = 0$
$\left(\frac{1}{i}\frac{\partial}{\partial y} + A' + B'\frac{\partial^2}{\partial x^2}\right)u = 0$
O
x

Figure 7

$$(26) \qquad k\rho \geq K \max \left(\frac{1}{\cos(\theta - \theta_o)}, \frac{1}{\cos(\theta - \theta + \pi)}, \frac{1}{\cos(\theta + \theta_{B'})} \right)$$

we can display an asymptotic development of (7) which does not require transition fields of the form (22).

$$(27a) \qquad u_s(\rho,\theta) = (R' - R)\, e^{ik(x \cos \theta_o + y \sin \theta_o)}$$

$$+ \frac{P^+(k \cos \theta)}{\cos \theta - \cos \theta_o} \; \frac{Z'(k \cos \theta_o)}{Z'(k \cos \theta)} \; \frac{\sin \theta \;\; e^{i(k\rho + \frac{\pi}{2})}}{\sqrt{2\pi k \rho}} \, .$$

$$+ O\left[(k\rho)^{-3/2}\right] \qquad \text{for} \qquad 0 \leq \theta < \theta_o$$

$$(27b) \qquad u_s(\rho,\theta) = \frac{Z'(k \cos \theta) \sin \theta}{P^+(-k \cos \theta)\, Z(k \cos \theta)(\cos \theta - \cos \theta_o)} \; \frac{e^{i(k\rho + \frac{\pi}{2})}}{\sqrt{2\pi k\rho}}$$

$$+ O\left[(k\rho)^{-3/2}\right], \; \theta_o < \theta \leq \pi \quad .$$

The complete solution is then displayed in figure 8 which illustrates the regions of validity.

5. Conclusion

We have found an explicit solution of an approximate formulation of a mathematically intractable problem. The solution is displayed in Figure 8 which illustrates the regions of validity. For convenience we list the symbols and notations that appear in that figure:

$$A = k/k_1$$

$$A' = k/k_2$$

$$B = A\left[1 - \sqrt{1 - \frac{A^2}{A^2+1}}\right]$$

$$B' = A'\left[1 - \sqrt{1 - \frac{A'^2}{A'^2+1}}\right]$$

$$R = \frac{\sin\theta_o - (A - B\cos^2\theta_o)}{\sin\theta_o + (A - B\cos^2\theta_o)}$$

$$R' = \frac{\sin\theta_o - (A' - B'\cos^2\theta_o)}{\sin\theta_o + (A' - B'\cos^2\theta_o)}$$

$$Z\,(k\cos\theta) = \sin\theta_o + (A - B\cos^2\theta_o)$$

$$Z'(k\cos\theta) = \sin\theta_o + (A' - B'\cos^2\theta_o)$$

$$P^+(k\cos\theta) = \sqrt{\frac{B'}{B}}\exp\left[-\frac{1}{\pi}\int_1^\infty \frac{1}{\cos\theta + \zeta}\tan^{-1}\frac{\sqrt{\zeta^2-1}\left[(A-A') + \zeta^2(B-B')\right]}{(\zeta^2-1)+(A'-B'\zeta^2)(A-B\zeta^2)}\,d\zeta\right]$$

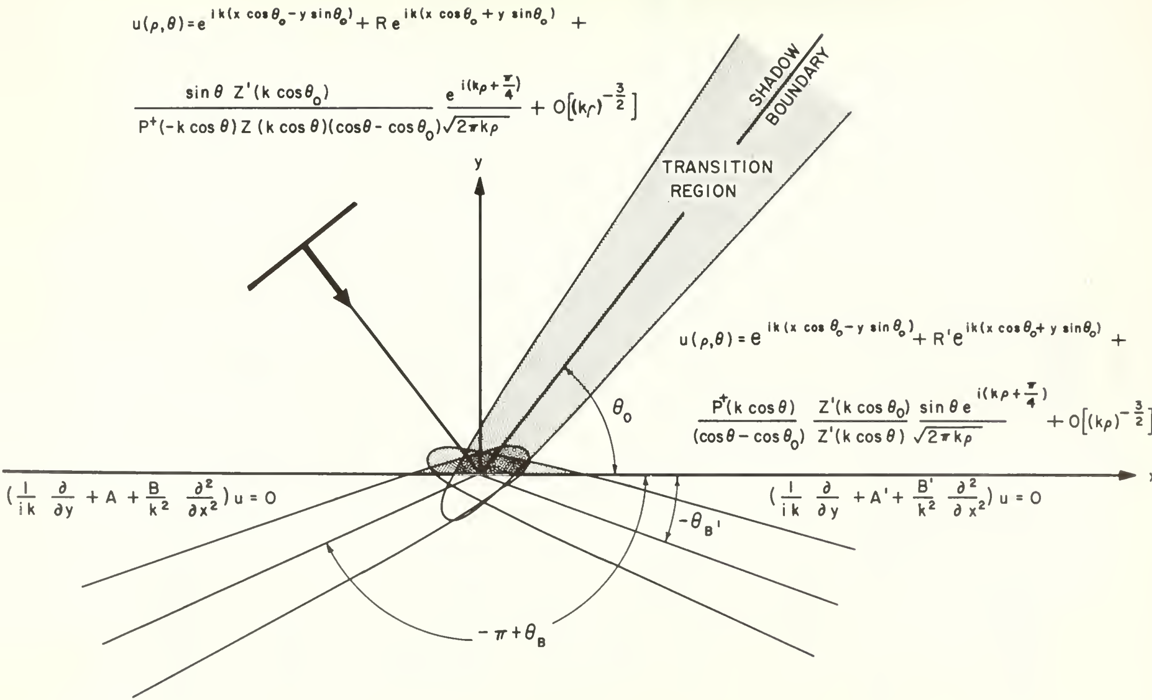
$u(\rho,\theta)=e^{ik(x\cos\theta_o-y\sin\theta_o)}+R\,e^{ik(x\cos\theta_o+y\sin\theta_o)}+$
$\frac{\sin\theta\, Z'(k\cos\theta_0)}{P^+(-k\cos\theta)Z(k\cos\theta)(\cos\theta-\cos\theta_0)}\frac{e^{i(k\rho+\frac{\pi}{4})}}{\sqrt{2\pi k\rho}}+O[(k\rho)^{-\frac{3}{2}}]$
SHADOW BOUNDARY
TRANSITION REGION
y
$u(\rho,\theta)=e^{ik(x\cos\theta_o-y\sin\theta_o)}+R'e^{ik(x\cos\theta_o+y\sin\theta_o)}+$
$\frac{P^+(k\cos\theta)}{(\cos\theta-\cos\theta_0)}\frac{Z'(k\cos\theta_0)}{Z'(k\cos\theta)}\frac{\sin\theta\, e^{i(k\rho+\frac{\pi}{4})}}{\sqrt{2\pi k\rho}}+O[(k\rho)^{-\frac{3}{2}}]$
θ_0
x
$(\frac{1}{ik}\frac{\partial}{\partial y}+A+\frac{B}{k^2}\frac{\partial^2}{\partial x^2})u=0$
$(\frac{1}{ik}\frac{\partial}{\partial y}+A'+\frac{B'}{k^2}\frac{\partial^2}{\partial x^2})u=0$
$-\theta_B{}'$
$-\pi+\theta_B$

Figure 8

APPENDIX A

Factorization of $K(\nu)$

We wish to show that the function

$$(1) \quad K(\nu) = \frac{k\sqrt{k^2 - \nu^2} + k^2 A' - B'\nu^2}{k\sqrt{k^2 - \nu^2} + k^2 A - B\nu^2}$$

can be expressed as

$$(2) \quad K(\nu) = P^+(\nu)\, P^+(-\nu)$$

in the cut ν-plane (cf. Fig. 4) (where $P^+(\nu)$ is analytic and zeroless for $\operatorname{Im} \nu > -\operatorname{Im} k$) by an appeal to the Cauchy Integral Theorem. For this purpose introduce

$$(3) \quad \ln D(\nu) = \ln \frac{B}{B'} K(\nu)$$

which is analytic in the strip $-\operatorname{Im} k < \operatorname{Im} \nu < \operatorname{Im} k$ since neither the numerator nor the denominator of $K(\nu)$ vanishes in that strip*. Furthermore since

$$\lim_{|\nu| \to \infty} \frac{B}{B'} K(\nu) = 1$$

we have

$$\lim_{|\nu| \to \infty} \ln D(\nu) = O\left(\frac{1}{\nu}\right)$$

---

*Cf. Appendix C, Part I.

so that by the Cauchy Integral Theorem we have

(4) $$\ell n\, D(\nu) = \frac{1}{2\pi i} \oint_C \frac{n\, D(\zeta)}{\zeta - \nu}\, d\zeta$$

where we take the contour C as shown in figure 9 where $\beta < k$. Define

(5) $$\ell n\, D^+(\nu) = \frac{1}{2\pi i} \int_{-\infty - i\beta}^{+\infty + i\beta} \frac{\ell n\, D(\zeta)}{\zeta - \nu}\, d\zeta$$

which is analytic and zeroless for Im $\nu > \beta$ and

$$|\ell n\, D^+(\nu)|$$

is bounded above and below - by positive constants in that half plane of regularity. Likewise we can define

$$\ell n\, D^-(\nu) = \frac{1}{2\pi i} \int_{-\infty + i\beta}^{+\infty + i\beta} \frac{\ell n\, D(\zeta)}{\zeta - \nu}\, d\zeta$$

which shares the regularity properties of $D^+(\nu)$ for Im $\nu < \beta$. We have

(6) $$\ell n\, D(\nu) = \ell n\, D^+(\nu) - \ell n\, D^-(\nu)$$

Since $D(\nu)$ is even in ν we have the relation

(7) $$\ell n\, D^+(\nu) = -\ell n\, D^-(-\nu)$$

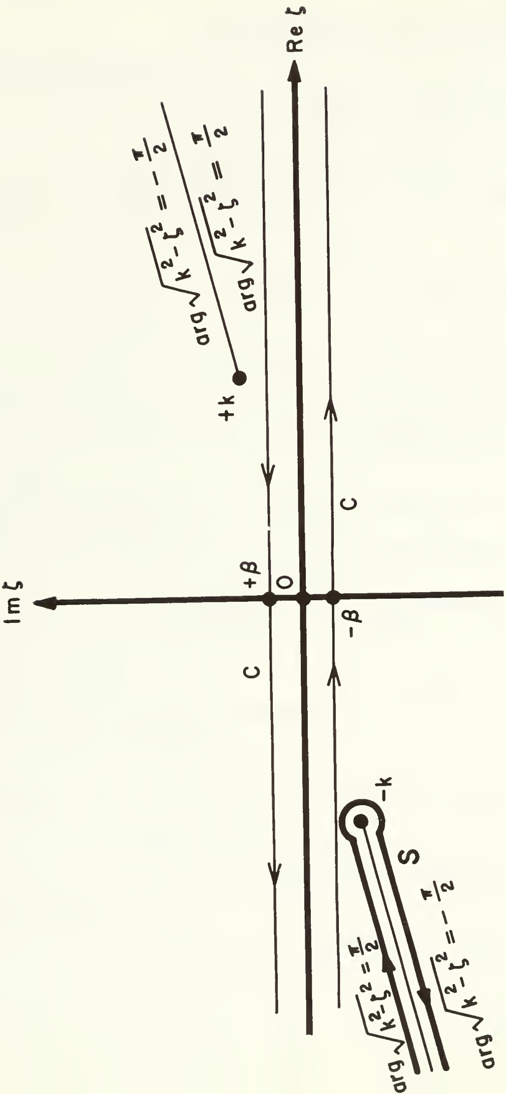
Re ζ
Im ζ
$\arg\sqrt{k^2-\zeta^2} = -\frac{\pi}{2}$
$\arg\sqrt{k^2-\zeta^2} = \frac{\pi}{2}$
+k
+β
O
-β
C
C
-k
S
$\arg\sqrt{k^2-\zeta^2} = \frac{\pi}{2}$
$\arg\sqrt{k^2-\zeta^2} = -\frac{\pi}{2}$

Figure 9

so that we can rewrite (4) as

$$\ell n\, D(\nu) = \ell n\, D^{+}(\nu) + \ell n\, D^{+}(-\nu)$$

or
$$D(\nu) = D^{+}(\nu)\, D^{+}(-\nu)$$

so that

$$K(\nu) = P^{+}(\nu)\, P^{+}(-\nu) \tag{8}$$

where

$$P^{+}(\nu) = \sqrt{\frac{B'}{B}}\, \exp\left[\frac{1}{2\pi i}\int_{-\infty\, -i\beta}^{+\infty\, -i\beta} \frac{\ell n\, \frac{B}{B'}\, K(\zeta)\, d\zeta}{\nu - \zeta}\right] . \tag{9}$$

For purposes of computation it is convenient to transform the contour defining $P^{+}(\nu)$ to obtain a representation which lends itself to numerical integration. For this purpose we can deform the contour $\mathrm{Im}\, \zeta = \beta$ to the path S as shown in figure 9 since $K(\zeta)$ is analytic and zeroless in the ζ-plane except for branch points at $\zeta = \pm k$. Using the same definitions of the $\arg \sqrt{k^2 - \zeta^2}$ as in the ν-plane we have $\arg \sqrt{k^2 - \zeta^2} = + \frac{\pi}{2}$ above the cut at $\zeta = -k$ and $\arg \sqrt{k^2 - \zeta^2} = - \frac{\pi}{2}$ below. It follows then that the integral that appears within the brackets of (9) can be rewritten

$$\frac{1}{2\pi i}\int_{-\infty}^{-k}\frac{1}{\zeta-\nu}\left[\ell n\,\frac{B}{B'}\left(\frac{+ki\sqrt{\zeta^2-k^2}+k^2A'-B'\zeta^2}{+ki\sqrt{\zeta^2-k^2}+k^2A-B\zeta^2}\right.\right.$$

$$\left.\left.-\ \ell n\,\frac{B}{B'}\ \frac{-ki\sqrt{\zeta^2-k^2}+k^2A'-B'\zeta^2}{-ki\sqrt{\zeta^2-k^2}+k^2A-B\zeta^2}\right)\right]\ d\zeta$$

which in turn can be expressed as

$$\frac{1}{\pi}\int_{-\infty}^{-k}\frac{1}{\zeta-\nu}\left[\tan^{-1}\frac{k\sqrt{\zeta^1-k^2}}{k^2A'-B'\zeta^2}-\tan^{-1}\frac{k\sqrt{\zeta^2-k^2}}{k^2A-B\zeta^2}\right]\ d\zeta$$

where the principle value x of the inverse tangent is taken between $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$ unless the argument of the inverse tangent is infinity in which case we move to the appropriate branch of $\tan^{-1} x$. We can simplify the integral still further by using the addition formula

$$\tan^{-1} x-\tan^{-1} y=\tan^{-1}\left[\frac{x-y}{1+xy}\right]$$

and replacing ξ by minus ξ to obtain

$$(10)\quad P^{+}(\nu)=\sqrt{\frac{B'}{B}}\ \exp-\left[\frac{1}{\pi}\int_{k}^{\infty}\frac{1}{\zeta+\nu}\ \tan^{-1}\frac{k\sqrt{\zeta^2-k^2}\left[k^2(A-A')+{}^2(B-B')\right]}{k^2(\zeta^2-k^2)+(k^2A'-B'\zeta^2)(k^2A-B\zeta^2)}\ d\zeta\right]\ .$$

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